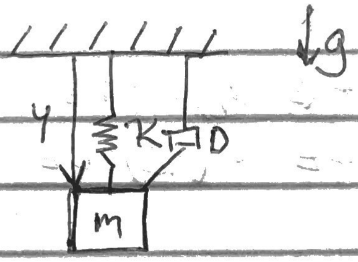
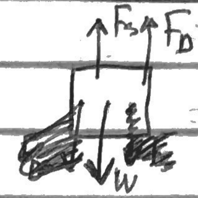
Dynamics of lumped Mechanical systems

Ex



FBD:



Newton's 2nd Law: $m\underline{a} = \Sigma \underline{F}$

* sign of force comes from direction of action

$$m\frac{d^2y}{dt^2} = W + F_s + F_D \Rightarrow m\frac{d^2y}{dt^2} = mg + F_s + F_D$$

Spring Force: F_s is a restoring force that acts to more object back to equilibrium. For small movement

$$\boxed{F_s} = +ky$$

Damping Force: F_D acts to slow an object down. At low velocities: $|F_D| = +D\frac{dy}{dt} \Rightarrow |F_D| = +D\dot{y}$

$$\frac{d^2y}{dt^2} \quad \frac{d}{dt}\left(\frac{dy}{dt}\right) = \frac{d\dot{y}}{dt}$$

$$\boxed{m\frac{d\dot{y}}{dt} = mg - ky - D\dot{y}}$$

Finding l_{eq}: At equilibrium, no motion or acceleration.

$$mg - k\,l_{eq} = 0$$

$$\boxed{l_{eq} = \frac{mg}{k}}$$

Movement about equilibrium:

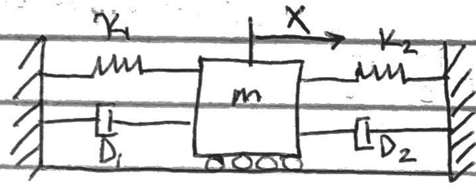
$y = z + l_{eq}$

$\dot{y} = \dot{z} \quad \Rightarrow \quad m\frac{d\dot{z}}{dt} = mg - k(z + l_{eq}) - D\dot{z}$

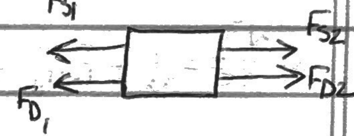
$= \cancel{mg} - kz - k\left(\cancel{\frac{mg}{k}}\right) - D\dot{z}$

$$\boxed{m\frac{d\dot{z}}{dt} = -kz - D\dot{z}}$$

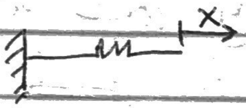
Ex



F.B.D.



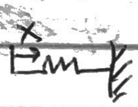
Spring 1:



Total stretch: $(x - 0)$

$|F_{s1}| = k_1(x - 0)$

Spring 2:



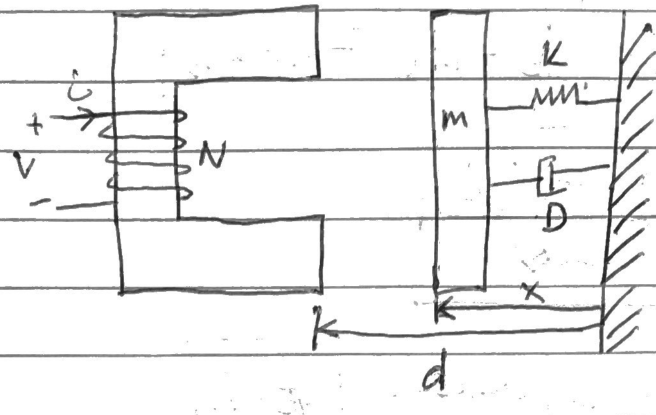
Total stretch: $(0 - x)$

$|F_{s2}| = k_2(0 - x)$

Same for dampers with $\dot{x}$

$m\frac{d\dot{x}}{dt} = k_2(0 - x) + D_2(0 - \dot{x}) - k_1(x - 0) - D_1(\dot{x} - 0) \Rightarrow \boxed{m\frac{d\dot{x}}{dt} = -(k_1 + k_2)x - (D_1 + D_2)\dot{x}}$

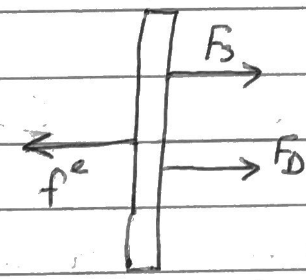
Ex



$$L(x) = \frac{\mu_0 A N^2}{2(d-x)}$$

$$f^e = \frac{1}{2} \frac{\mu_0 A N^2}{2(d-x)^2} i^2$$

F.B.D.



* By convention, f^e always points in the positive x direction. Always.

$$\boxed{m\frac{d\dot{x}}{dt} = f^e - kx - D\dot{x}}$$

$$V = \frac{d\lambda}{dt} \Rightarrow \boxed{V = L(x)\frac{di}{dt} + \frac{\partial L}{\partial x} i \dot{x}}$$

State variables: give current configuration of system and can be used to predict future configuration.

Mechanical: x, $\dot{x}$
Electrical: V, i

State space: Look for how state variables change over time.

Rewrite Newton's 2nd Law: $\boxed{\frac{d\dot{x}}{dt} = \frac{1}{m}(f^e - kx - D\dot{x})}$

Rewrite KVL: $\boxed{\frac{di}{dt} = \frac{V - \left(\frac{\partial L}{\partial x}\dot{x}\right)i}{L(x)}}$

Introduce 3rd equation for completeness: $\boxed{\frac{dx}{dt} = \dot{x}}$

State vector: $\underline{X} = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix}$

$x_1 = x$. or any permutation of this.

$x_2 = \dot{x}$

$x_3 = i$

$$\boxed{\begin{aligned} \frac{dx_1}{dt} &= x_2 \\ \frac{dx_2}{dt} &= \frac{1}{m}(f^e - kx_1 - Dx_2) \\ \frac{dx_3}{dt} &= \frac{V - \left(\frac{\partial L}{\partial x_1} x_2\right)x_3}{L(x_1)} \end{aligned}}$$

State space in general: $\boxed{\frac{d\underline{x}}{dt} = \underline{f}(\underline{x})}$